

Vibration source identification caused by bearing faults based on SVD-EMD-ICA

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Abstract: Aiming at the sensitivity to noises and the underdetermined number of signal in a single channel signal BSS (blind source separation), a high accuracy method of BSS was proposed through the proper combination of SVD(singular value decomposition), EMD(empirical model decomposition) and ICA(independent component analysis). The Hankel matrix, which was constructed through the single, was decomposed by SVD to get singular values series. The crucial singular values were selected based on the maximum value in different series to reconstruct the vibration signal. The reconstructed signal was decomposed by EMD to get IMFs (intrinsic model functions), which were regarded as the supplementary channels of ICA. FastICA was applied to execute the blind source separation on the condition of numbers ascertained by the principle called as Bayes information. The method was verified through a lot of simulation and test data about bearing faults, the result shows that the method could effectively isolate the vibration sources caused by different fault model.

Keywords: SVD;EMD;BSS; Fault model identification

I. Introduction

Bearing system has the interaction of inner ring, rolling body, outer ring and cage, there is generally a combined incentives of more failure modes of faulty bearing. Therefore, the fault source separation is the key to bearing fault detection correctly. But in practical engineering, the bearing vibration detection generally can only detect the vibration signals of the single channel due to space constraints. Multi-fault sources separation of bearing is a single-channel blind source separation problem.

However, the separation of single-channel signal is a typical underdetermined blind source separation Problem [1]. The number of sensor significantly less than the number of vibration source, to which many domestic and foreign scholars, respectively, has put forward many signal channel supplementary methods to solve the underdetermined separation, turning it into the general separation well-posed problem. As the literature[2] proposed a use of phase space reconstruction and singular value decomposition to realize the underdetermined blind source separation, the method has large human factors in selecting singular value during the inverse transformation. Literature[3] presented a blind separation method based on wavelet packet decomposition of the related mechanical vibration source, the decomposition order of wavelet packet affects the separation result of blind source separation. Literature[4] studied the underdetermined blind source separation deeply based on sparse representation, and [5] proposed the median clustering algorithm to implement the underdetermined blind source separation, the two methods are based on the representation of source signal, the separation effect is not ideal when the source signal has weak sparsity. In order to decrease the uncertainty of analysis results which is brought by sparse decomposition in enhancing signal channel, the empirical mode

decomposition was introduced to the single-channel signal blind source separation in literature[6, 7], which can decompose the non-stationary signal into a series of stable and stationary signals of single mode, and the decomposed multi-scale signals can be used as virtual channel of blind source separation, which can successfully separate the bearing and gear fault vibration source. But the precision of empirical mode decomposition and the correctness of modes are affected greatly by the noise[8].

However, the noise is inevitable in practical engineering. To make full use of the adaptive feature of EMD in signal channel supplementary in blind source separation, and eliminate the defect of EMD to noise. The SVD is introduced to the study of EMD and BSS, the paper proposes a method for single-channel BSS based on SVD-EMD-ICA. Hankel matrix is constructed for single-channel signal and singular value decomposition of the matrix, make use of the singular values difference spectrum unilateral maximum principle as the key selection criterion to improve the signal-to-noise ratio of the reconstructed signal, EMD is applied to the reconstructed signal to get multi-scale intrinsic mode functions, which can be used as a signal enhanced channel, and the Bayesian criterion is used to estimate the source number, archive the single-channel BSS by FastICA algorithm. Finally, the simulation data and experimental data for the verification of the proposed method and model. The paper is organized as follows. In Section 2, a brief introduction of the proposed method, including SVD and EMD. After the presented recognition model in Section 3, a experimental verification, including simulation data and experimental data, of the presented model is carried on in Section 4, while its effectiveness is evaluated in comparison with the EMD-ICA method. Conclusion is presented in Section 5.

II. BSS method based on SVD-EMD-ICA

2.1 De-noising principle of SVD based on Hankel matrix

Supposed the analysis signal $x = [x(1), x(2), \dots, x(N)]$, in which N is the length of signal, constructing a $m \times n$ dimensional Hankelmatrix [9] as followed

$$H = \begin{bmatrix} x(1) & x(2) & \cdots & x(n) \\ x(2) & x(3) & \cdots & x(n+1) \\ \vdots & \vdots & \vdots & \vdots \\ x(m) & x(m+1) & \cdots & x(N) \end{bmatrix} \quad (1)$$

where $1 < n < N$, $m = N - n + 1$.

For a real matrix $H \in R^{m \times n}$, regardless of whether its row-column is relevant, there must be an orthogonal matrix $U \in R^{m \times m}$ and an orthogonal matrix $V \in R^{n \times n}$ satisfy the formula as followed

$$H = USV^T \quad (2)$$

where $S = (diag(\sigma_1, \sigma_2, \dots, \sigma_q), \mathbf{0}) \in R^{m \times n}$

in which $\mathbf{0}$ represents zero matrix. $q = \min(m, n)$, and $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_q$, formula (2) is the singular value decomposition, and $\sigma_i (i = 1, 2, \dots, q)$ are the singular values of matrix H [10].

The nature of SVD is that the original signal is decomposed into the simple linear superposition of a series of component signals, which have a zero phase shift characteristics. The signal containing noise $x(i)$ is

represented as

$$x(i) = s(i) + \xi(i) \quad i = 1, 2, \dots, N \quad (3)$$

where $s(i)$ is the pure signal, and $\xi(i)$ is the noise signal. The Hankel matrix H constructed by $x(i)$ can be represented as[11]

$$H = H_s + H_\xi \quad (4)$$

where H_s and H_ξ , respectively, represents the Hankel matrix constructed by $s(i)$ and $\xi(i)$

$$H_s, H_\xi \in R^{m \times n}.$$

Seen from the construction feature of Hankel matrix: there is only a point data difference of the two adjacent rows, for the Hnakel matrix H_s of pure signal $s(i)$, the two adjacent rows are highly correlation, is a kind of morbid matrix. The feature of its singular value distribution is that the first k singular values are bigger, and the others are close to zero, the singular value distribution curve has a larger catastrophe at the point k ; for the Hankel matrix H_ξ of noise signal $\xi(i)$, although its two adjacent rows differ by just one point data, but between noise signal is relatively independent, so the matrix is the full rank matrix, and the characteristics of singular value distribution is that all of the singular values were similar and small volatile. Therefore, the key to the separation of pure signal and noise signal is a reasonable choice of singular value catastrophe point. Base on literature[12] the singular value difference spectrum unilateral maximum principle is used to determine the singular value catastrophe point.

2.2 BSS based on EMD virtual channel

EMD is carried on to the de-noised signal by using SVD, the nonlinear and non-stationary signal is decomposed into a series of linear and stationary intrinsic mode functions adaptively, the IMFs must meet the following two conditions: 1)within the whole data set, the number of extrema(including the maximum points and the minimum points) must be either equal to the crossing-zero points or differ at most by one; 2)at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. The specific decomposition process, see[13].

EMD is applied to the single-channel observation signal $x(i)$, get its IMFs and a residual r_n , $x_{im} = (c_1, c_2, \dots, c_n, r_n)^T$. Then, the single-channel observation signal $x(i)$ and its IMF components are formed into a new multidimensional observation signal $x_{inf} = (x_1, c_1, c_2, \dots, c_n, r_n)^T$, the underdetermined BSS problem is transformed into well-posed BSS, using Bayesian information criterion to estimate the source number[14]. Finally, FastICA achieve source separation.

III. Recognition model based on SVD-EMD-ICA

The vibration source recognition model based on SVD-EMD-ICA is shown in Fig.1.

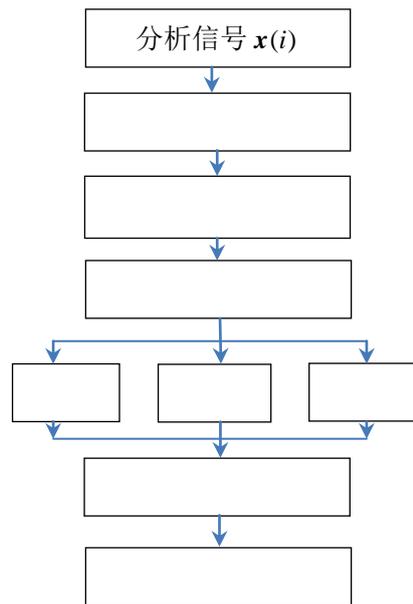


Fig.1 vibration source recognition model

The main steps are as follows:

Step1: SVD de-noising on single-channel observation signal. Using SVD to decompose single-channel observation signal based on Hankel matrix, and singular value difference spectrum unilateral maximum principle is used to determine the effective noise reduction order, eliminate noise effects.

Step2: Estimate signal source. The EMD decomposition of de-noised signal, then, the de-noised signal and its IMFs are formed into a new multidimensional signal, using the Bayesian information criterion to estimate the number of signal source.

Step3: Reconstruction of multi-channel observation signal. According to estimated source number, the de-noised signal and its IMFs are formed into a new multi-channel signal whose dimension is equal to the estimated source number.

Step4: BSS of multi-channel signal. Making use of FastICA algorithm to achieve BSS of multi-channel signal, get the independent sources.

IV. Experimental verification of recognition model

4.1 Verification of simulation signal

In order to verify the effectiveness and feasibility of the proposed method, construct three mechanical signals and a white noise signal as follows:

$$s_1(t) = \sin(2\pi f_1 t)(1 + \beta \sin(2\pi f_r t))$$

$$s_2(t) = \cos(2\pi f_2 t - 10)$$

$$s_3(t) = \sin(2\pi f_3 t) \cos(2\pi f_4 t)$$

$$s_4(t) = randn(t)$$

$$f_1 = 100\text{Hz} \quad , \quad f_2 = 50\text{Hz}$$

where $f_3 = 10\text{Hz} \quad , \quad f_4 = 20\text{Hz}$

$$f_r = 20\text{Hz} \quad , \quad \beta = 0.5$$

Sampling frequency $f_s = 1024\text{Hz}$, sampling number $N = 1024$. The time domain and frequency domain waveform of source signal are shown in Fig.2.

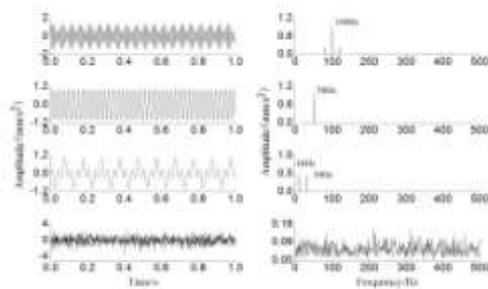


Fig.2 the time domain and frequency domain waveform of source signal

In order to obtain single-channel observation signal, the three mechanical source signals and a white noise signal are mixed by the model as follow:

$$x_s(t) = a_1s_1(t) + a_2s_2(t) + a_3s_3(t) + s_4(t) \tag{5}$$

where x_s is the single-channel mixed signal, a_1 , a_2 and a_3 are superposition factors, s_4 is random white noise.

The constructed random matrix is $A = [a_1 \ a_2 \ a_3]$ in which $a_1 = 0.9915$, $a_2 = 0.8153$ and $a_3 = 0.6182$.

The time domain and frequency domain waveform of the single-channel mixed signal are shown in Fig.3.

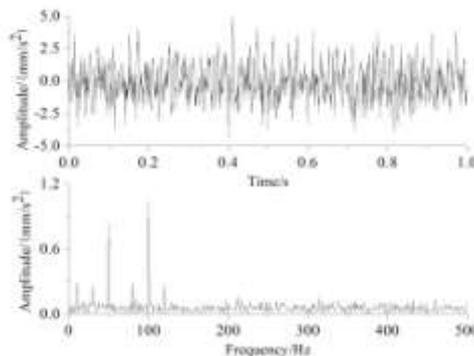


Fig.3 time domain and frequency domain waveform of single-channel mixed signal

By using the method of Hankel matrices reconstructing phase space, the single-channel signal $x_s(t)$ is decomposed by SVD, and select the embedding dimension $m = 512$. In order to facilitate clear observation, selecting the first 100 points of the singular value sequences and the singular value difference spectrum sequences to be amplified, as shown in Fig.4. It can be seen that the singular value difference spectrum effectively reflects the number of singular value contains large amounts of information in which the differential values corresponding to the singular value order of 2, 4, and 12 are large. Selecting the effective order of 12 to reduce noise according to the unilateral maximum principle, and the result of noise reduction is shown in Fig.5.

As can be seen from Fig.5 corresponding to the maximum spectral peak point to a valid order of noise reduction, lost part of useful information of the source signal, at the same time, it can be seen from Tab.1 that the noise reduction effect of unilateral maximum is significantly better than maximum spectral peak.

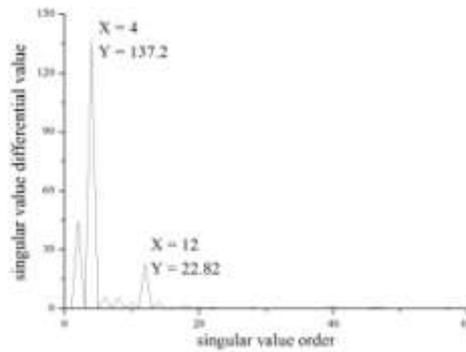


Fig.4 singular value sequences and singular value difference spectrum

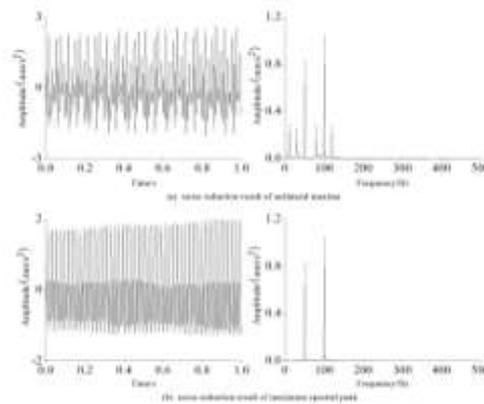


Fig.5 noise reduction results of single-channel observation signal

Tab.1 SNR and MSE of single-channel signal before and after noise reduction

signal types	SNR/dB	MSE
single-channel signal	-0.0024	0.9814
de-noised signal of unilateral maxima	16.1118	0.0240
de-noised signal of maximum spectral peak	7.7666	0.1640

The estimated source number is 3 by applying Bayesian information principle. The de-noised signal $x_1(t)$ of unilateral maxima and its first two IMF components are formed into a new multi-channel observation signal $x = (x_1, c_1, c_2)^T$. Using FastICA algorithm for blind source separation of multi-channel observation signal, the time domain and frequency domain waveform of the obtained separated signals are shown in Fig.6. But without noise reduction processing by SVD, carry on EMD directly to the single-channel signal, the blind source

separation result of the reorganized multi-channel observation signal is shown in Fig.7.

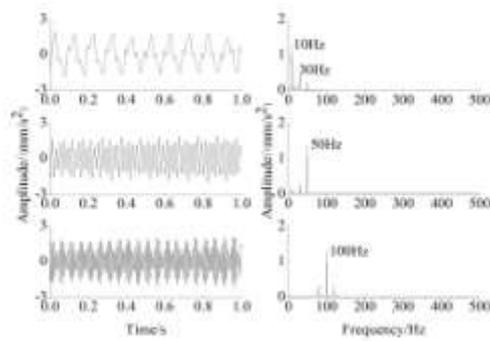


Fig.6 separation result of SVD-EMD-ICA

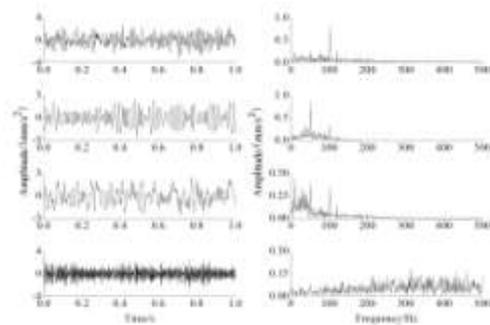


Fig.7 separation result of EMD-ICA

By comparing Fig.2, Fig.6 and Fig.7, it can be seen , as shown in Fig.6, the time domain waveform and spectrum distribution of three separated signals have little difference with the three simulation signals shown in Fig.2, the separation results are good; while, as shown in Fig.7, more noise interference exist in the separated signals. Thus, the separation effect of proposed method in the paper is better than the EMD-ICA method.

4.2 Validation of experimental signal

In order to further verify the validity of SVD-EMD-ICA, the method is applied to rolling bearing fault signal measured. Data is from the US Case Western Reserve University in electrical engineering lab of rolling bearing fault simulation test bench[14]. The faulty bearing type is SKF6205, using EDM technology processing a single point of failure in bearing, the motor speed in the experiment is 1797RPM, that is to say the rotation frequency is 29.95Hz, bearing failure is 0.1778mm in diameter and 0.2794mm in depth of the outer ring failure. The calculated fault characteristic frequency of bearing outer ring is 107.36Hz. The sampling frequency is 12KHz, take 2048 sampling points, the time domain waveform and spectrum distribution are shown in Fig.8.

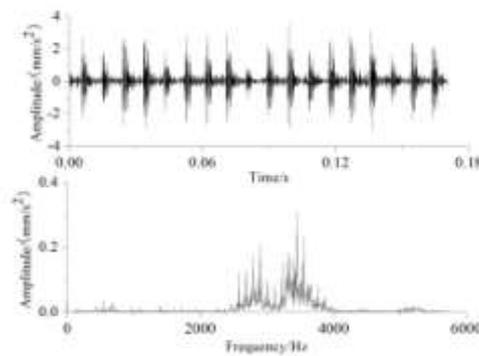


Fig.8 time domain and frequency domain waveform of bearing outer ring fault signal

The time domain waveform and frequency domain distribution of the separated signals are shown in Fig.9 which are obtained by applying SVD-EMD-ICA to the separation of bearing outer ring fault data, Fig.10 represents the part of spectrum distribution of separated signal, from which can be seen at the frequency points 99.6Hz, 509.8Hz, 744.1Hz, 1042.9Hz, 1154.3Hz, 1260.0Hz and 1798.8Hz there exist obvious spectral peak, those frequencies basically tallies with bearing outer ring fault characteristic frequency and its frequency doubling as shown in Tab.2. Thus, it can be seen that the bearing outer ring fault feature is more obvious by applying blind source separation.

Tab.2 frequencies of separated signal corresponding to outer ring fault characteristic frequency

fault characteristic frequency/Hz	frequency of separated signal/Hz	theoretical frequency/Hz
5 octave	509.8	536.8
7 octave	744.1	751.5
10 octave	1042.9	1073.6
11 octave	1154.3	1180.9
12 octave	1260.0	1288.3
17 octave	1798.8	1825.1

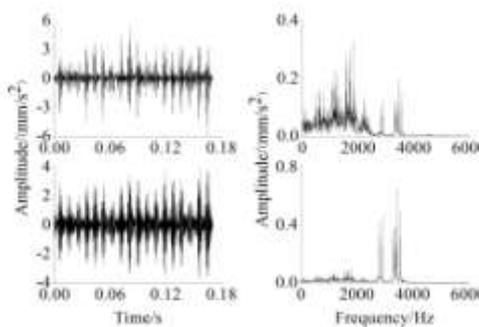


Fig.9 time domain and frequency domain of separated signal

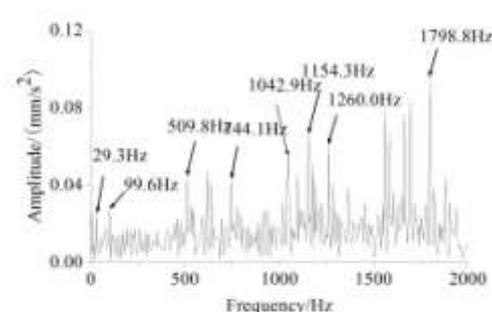


Fig.10 part of spectrum of separated signal

V. Conclusions

- 1) Organic integration of SVD, EMD and ICA, using the noise reduction function of SVD, and EMD can convert non-stationary model into stable vibration mode, and the source separation ability of ICA, has realized the blind source separation of single-channel noised signal.
- 2) The proposed method is applied to bearing vibration source identification study, successfully recognized the fault vibration characteristics of the bearing outer ring, and adapt to the bearing fault signal analysis.

VI. Acknowledgement

The research is supported by the National Natural Science Foundation of China (Grant No.51305358), and Shanghai University of Engineering Science Innovation Fund for Graduate Students (Grant No.14KY1007).

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